2008

Mathematics Curriculum Guide

Catholic Diocese of Wilmington, Delaware

Algebra II Standards

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Mission

The Catholic school has the responsibility to prepare all students to function effectively in today's society and to bring Christian values to their world. Integral to the complete formation of the child in our Catholic schools is the study of Mathematics. Students of the twenty-first century must be taught to value Mathematics and become competent and confident in reasoning, making connections, and communicating in order to be better problem solvers. They should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibility.

Vision

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

Philosophy

Mathematics is learned through an approach that begins with concrete explorations and leads students to an understanding of symbolic representations. All students must have equal access to rigorous, high quality instruction to become mathematically literate. The uniqueness of each student should be nurtured by using differentiated strategies in response to various learning styles. A broad variety of assessments must provide multiple indicators of student achievement.

Communicating mathematically enables students to solve problems by acquiring information through reading, listening, and observing. Students will be able to translate information into mathematical language and symbols, process the information mathematically, and present the results in written, oral, and visual formats to demonstrate their mathematical literacy.

Students achieve mastery of computational skills through the employment of age-appropriate materials while also developing higher-level critical thinking skills. In our progressively changing world, students need to know how to properly utilize innovative tools, media, and technology to solve cross-curricular mathematical problems. Technology, however, is not a replacement for the comprehension of mathematical concepts.

The Mathematics program prepares students to fulfill personal ambitions and career goals in an ever changing world. Classrooms that encourage investigation, collaboration, and

resourcefulness in the problem solving process empower students beyond the classroom. It is through the cornerstones of communication, teamwork, and opportunity that we instill into our students a deeper appreciation and knowledge of mathematics so that they may become productive Catholic citizens of the world.

Goals

All students will:

1. Learn to appreciate mathematics, reason mathematically, and communicate mathematically.
2. Utilize their mathematical skills to become competent problem solvers.
3. Make mathematical connections to real life situations and to other areas of the curriculum.
4. Use technology appropriately and effectively.
5. Apply ethical and critical thinking.

Expectations for Learning

We commit to the following expectations:

1. That all grade levels students:
    - Learn to think critically, logically, ethically, and analytically
    - Learn to express ideas orally and in writing using correct mathematical terminology
    - Learn to apply the techniques of mathematics to real world situations
    - Understand that mathematics is important to function in today's world
    - Utilize technology responsibly
1. That computers, calculators, manipulatives and other tools of learning should be used routinely as an integral part of both instruction and assessment.
2. That mathematics teachers be encouraged to participate in professional development activities.
3. That mathematics coordinators hold regularly scheduled faculty meetings to facilitate communication and to analyze the strengths and weaknesses within the program.
4. That the teacher utilize the mathematics curriculum guidelines for grade level instruction.
5. That teachers provide differentiated instruction and assessment.

ALGEBRA II

***As life-long learners**, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.*

The Diocese of Wilmington has established the following mathematics Standards to clarify for teachers, students, and parents the knowledge, understanding, and skills students should attain in ALGEBRA II:

Standard 1 — Relations and Functions
Students recognize and graph polynomial, rational, and algebraic functions. They understand the concept of functional notation and use it to combine functions by composition. They solve equations and inequalities by examining their graphs and interpret situations as functions in graphs, formulas, and words.

Standard 2 — Linear and Absolute Value Equations and Inequalities
Students graph linear equations and inequalities involving absolute value. They use a variety of methods to solve systems of up to three linear equations in up to three variables, and they model data with linear equations and make predictions from the results.

Standard 3 — Quadratic Equations and Functions
Students extend the number system by defining complex numbers, relating them to the real numbers, and using them to solve quadratic equations using the quadratic formula, by factoring, and completing the square. They draw graphs of quadratic functions and apply transformations to the functions. They find and interpret zeros and maximum and minimum values, and solve word problems. They also solve equations containing radicals and solve pairs of equations.

Standard 4 — Polynomials
Students understand and use the binomial theorem for positive integer powers. They learn techniques for factoring polynomials in order to solve equations and related word problems. They find approximate solutions of equations using graphing technology and write equations with given solutions. They understand the relationships among the solutions of an equation, the zeros of a function, the *x*-intercepts of a graph, and the factors of a polynomial.

Standard 5 — Algebraic Fractions
Students understand and use the concepts of negative and fractional exponents. They add, subtract, multiply, divide, and\ simplify algebraic fractions. They solve equations involving algebraic fractions and solve related word problems. They also solve problems of direct, inverse, and joint variation.

Standard 6 — Logarithmic and Exponential Functions
Students understand the concepts of logarithmic and exponential functions. They graph exponential functions and solve problems of growth and decay. They understand the inverse relationship between exponents and logarithms and use it to prove laws of logarithms and to solve equations. And they convert logarithms between bases and simplify logarithmic expressions.

Standard 7 – Trigonometry
Students understand the concept of trigonometric functions as they relate to triangles. They use trigonometric identities, sums and difference formulas, inverse trigonometric functions, and radians and degree measurements for angles. They also use law of sines and law of cosines to find sides and angles of triangles. They evaluate, verify, and graph trigonometric functions.

Standard 8 – Matrices
Students understand the concept of matrices. They add, subtract, and multiply with matrices. They also use inverse and identity matrices to solve problems. Students find determinants and inverses of matrices. They use matrices to solve systems and related word problems.

Standard 9 — Mathematical Reasoning and Problem Solving
In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use counterexamples to show that a general statement is false.

Students should also develop the following learning skills by Grade 12 that are integrated throughout the National Council of Teachers of Mathematics (NCTM) Standards:

Communication
As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Reasoning and Proof
Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Connections
Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Representation
Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Standard 1

Relations and Functions

Students graph relations and functions and find zeros. They use function notation and combine functions by composition. They interpret functions in given situations.

A2.1.1 Recognize and graph various types of functions, including polynomial, rational, and algebraic functions.

Example: Draw the graphs of the functions $y = x^4 + x^2$, $y = \frac{7}{x-2}$, and $y = \sqrt{x+2}$.

A2.1.2 Use function notation. Add, subtract, multiply, and divide pairs of functions.

Example: Let $f(x) = 7x + 2$ and $g(x) = x^2$. Find the value of $f(x) \bullet g(x)$.

A2.1.3 Understand composition of functions and combine functions by composition.

Example: Let $f(x) = x^3$ and $g(x) = x - 2$. Find $f(g(x))$.

A2.1.4 Graph relations and functions with and without graphing technology.

Example: Draw the graph of $y = x^3 + 3x^2 - x + 3$.

A2.1.5 Find the zeros of a function.

Example: In the last example, find the zeros of the function; i.e., find x when $y = 0$.

A2.1.6 Solve an inequality by examining the graph.

Example: Find the solution for $x^3 + 3x^2 - x + 3 < 0$ by graphing $y = x^2 + x - 2$.

A2.1.7 Graph functions defined piece-wise.

Example: Sketch the graph of $f(x)\begin{cases} x+2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x > 0 \end{cases}$.

A2.1.8 Interpret given situations as functions in graphs, formulas, and words.

Example: You and your parents are going to Boston and want to rent a car at Logan International Airport on a Monday morning and drop the car off in downtown Providence, R.I., on the following Wednesday. Find the rates from two national car companies and plot the costs on a graph. Decide which company offers the best deal. Explain your answer.

Standard 2

Linear and Absolute Value Equations and Inequalities

Students solve systems of linear equations and inequalities and use them to solve word problems. They model data with linear equations.

A2.2.1 Graph absolute value equations and inequalities.

Example: Draw the graph of $y = 2x - 5$ and use that graph to draw the graph of $y = |2x - 5|$.

A2.2.2 Use substitution, elimination, and matrices to solve systems of two or three linear equations in two or three variables.

Example: Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$.

A2.2.3 Use systems of linear equations and inequalities to solve word problems.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

A2.2.4 Find a linear equation that models a data set using the median fit method and use the model to make predictions.

Example: You light a candle and record its height in centimeters every minute. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.5), (3, 14.8), (4, 13.2), (5, 11.5), (6, 10.0), (7, 8.2), (9, 4.9), and (10, 3.1). Find the median fit line to express the candle's height as a function of the time and state the meaning of the slope in terms of the burning candle.

Standard 3

Quadratic Equations and Functions

Students solve quadratic equations, including the use of complex numbers. They interpret maximum and minimum values of quadratic functions. They solve equations that contain square roots.

A2.3.1 Define complex numbers and perform basic operations with them.

Example: Multiply $7 - 4i$ and $10 + 6i$.

A2.3.2 Understand how real and complex numbers are related, including plotting complex numbers as points in the plane.

Example: Plot the points corresponding to $3 + 2i$ and $1 - 4i$. Add these complex numbers and plot the result. How is this point related to the other two?

A2.3.3 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ in complex numbers.

A2.3.4 Graph quadratic functions. Apply transformations to quadratic functions. Find and interpret the zeros and maximum or minimum value of quadratic functions.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

A2.3.5 Solve word problems using quadratic equations.

Example: You have 100 feet of fencing to make three sides of a rectangular area using an existing straight fence as the fourth side. Construct a formula in a spreadsheet to determine the area you can enclose and use the spreadsheet to make a conjecture about the maximum area possible. Prove (or disprove) your conjecture by solving an appropriate quadratic equation.

A2.3.6 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x+9} = 9 - \sqrt{x}$.

A2.3.7 Solve pairs of equations, one quadratic and one linear or both quadratic.

Example: Solve the system of equations $y = x^2 - 5x + 1$, $x + y + 2 + 0$.

Standard 4

Polynomials

Students use the binomial theorem, divide and factor polynomials, and solve polynomial equations.

A2.4.1 Understand the binomial theorem and use it to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$

A2.4.2 Divide polynomials by others of lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

A2.4.3 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 - 27 = 0$ by factoring.

A2.4.4 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

A2.4.5 Use polynomial equations to solve word problems.

Example: You want to make an open-top box with a volume of 500 square inches from a piece of cardboard that is 25 inches by 15 inches. Find the possible dimensions of the box.

A2.4.6 Write a polynomial equation given its solutions.

Example: Write an equation that has solutions $x = 2$, $x = 5i$ and $x = -5i$.

A2.4.7 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the *x*-intercepts of a graph, and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

Standard 5

Algebraic Fractions

Students use negative and fractional exponents. They simplify algebraic fractions and solve equations involving algebraic fractions. They solve problems of direct, inverse, and joint variation.

A2.5.1 Understand and use negative and fractional exponents.

Example: Simplify $(2a^{-2}b^3)^4(4a^{-3}b^{-1})^{-2}$.

A2.5.2 Add, subtract, multiply, divide, and simplify algebraic fractions.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$.

A2.5.3 Simplify complex fractions.

Example: Simplify $\left(\frac{5}{x-2}+\frac{2}{x+3}\right) \div \left(\frac{1}{x+3}+\frac{7}{x-2}\right)$.

A2.5.4 Solve equations involving algebraic fractions.

Example: Solve $\frac{10}{n}+\frac{5}{n^2-4}=\frac{7}{n-2}$.

A2.5.5 Solve word problems involving fractional equations.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together?

A2.5.6 Solve problems of direct, inverse, and joint variation.

Example: One day your drive to work takes 10 minutes and you average 30 mph. The next day the drive takes 15 minutes. What is your average speed that day?

Standard 6

Logarithmic and Exponential Functions

Students graph exponential functions and relate them to logarithms. They solve logarithmic and exponential equations and inequalities. They solve word problems using exponential functions.

A2.6.1 Graph exponential functions.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

A2.6.2 Prove simple laws of logarithms.

Example: Use the fact that $a^x \bullet a^y = a^{x+y}$ to show that $\log_a(pq) = \log_a p + \log_a q$.

A2.6.3 Understand and use the inverse relationship between exponents and logarithms.

Example: Find the value of $\log_{10}(10^7)$.

A2.6.4 Solve logarithmic and exponential equations and inequalities.

Example: Solve the equation $\log_2 x = 5$.

A2.6.5 Use the definition of logarithms to convert logarithms from one base to another.

Example: Write $\log_{10} 75$ as a logarithm to base 2.

A2.6.6 Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values.

Example: Simplify $\log_3 81$.

A2.6.7 Use calculators to find decimal approximations of natural and common logarithmic numeric expressions.

Example: Find a decimal approximation for ln 500.

A2.6.8 Solve word problems involving applications of exponential functions to growth and decay.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

Standard 7

Trigonometry

Students use trigonometric functions to solve problems involving triangles.

A2.7.1 Use trigonometric relationships to evaluate trigonometric functions.

Example: Approximate the value of $\tan 72^\circ$ to three decimal places.

A2.7.2 Measure angles using degrees and radians.

Example: Find the angle between the hands of a clock at 4:00. Give your answer in radians and degrees.

A2.7.3 Evaluate trigonometric functions of any angle.

Example: In the last example, find the sum of the first 10 terms.

A2.7.4 Evaluate inverse trigonometric functions.

Example: Solve the equation: $\sin\theta = 0.7431$ when $90^o < \theta < 180^o$

A2.7.5 Use law of sines and law of cosines to find sides and angles of a triangle.

Example: A city park is in the shape of a triangle with side lengths of 10 meters, 12 meters, and 14 meters. Find the area of the park.

A2.7.6 Graph trigonometric functions.

Example: Graph the following function. $y = -4\sin 6\pi x$.

A2.7.7 Verify trigonometric equations.

Example: Simplify the expression $\frac{\sin(-x)}{\csc(-x)} + \sin^2 x$.

Standard 8

Matrices

Students use matrices and matrix operations.

A2.8.1 Use matrix operations to simplify matrices.

Example: $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 4 & -1 & -2 \\ 0 & 3 & 6 \end{bmatrix}$ Simplify. $\begin{bmatrix} 2 & 8 \\ 9 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 7 \\ 1 & 5 \end{bmatrix}$

A2.8.2 Multiply matrices.

Example: Find $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 4 & -1 & -2 \\ 0 & 3 & 6 \end{bmatrix}$.

A2.8.3 Find determinants and use Cramer's Rule.

Example: Evaluate the determinant $\begin{bmatrix} 4 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & 1 & 5 \end{bmatrix}$

A2.8.4 Find the inverse of matrices and use the identity matrix.

Example: Find the inverse. $\begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$

A2.8.5 Solve systems using Inverse Matrices.

Example: Solve the system: 2x – 3y = 15 and 8x + 7y = -11.

Standard 9

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

A2.9.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: The swimming pool at Roanoke Park is 24 feet long and 18 feet wide. The park district has determined that they have enough money to put a walkway of uniform width, with a maximum area of 288 square feet, around the pool. How could you find the maximum width of a new walkway?

A2.9.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 20 feet. Is that reasonable?

Students develop and evaluate mathematical arguments and proofs.

A2.9.3 Decide if a given algebraic statement is true always, sometimes, or never (Statements involving rational or radical expressions or logarithmic or exponential functions).

Example: Is the statement $(a^x)^y = a^{xy}$ true always, sometimes, or never?

A2.9.4 Use the properties of number systems and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$, explaining why you can take each step.

A2.9.5 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $\sqrt{x+6} = x$ comes up with the solution set {-2, 3}. Explain why {-2, 3} is not the solution set to this equation, and why the "check" step is essential in solving the equation.

A2.9.6 Use counterexamples to show that statements are false.

Example: Show by an example that this statement is false: The product of two complex numbers is never a real number.